

Indian Statistical Institute, Bangalore Centre.
Mid-Semester Exam : Graph Theory

Instructor : Yogeshwaran D.

Date : February 27th, 2019.

Max. points : 15.

Time Limit : 1.5 hours.

Answer any three questions only. All questions carry 5 points.

Give necessary justifications and explanations for all your arguments. If you are citing results from the class or assignments, mention it clearly. See the end of the question paper for notations.

1. If G is a simple graph, show that $diam(G) \geq 3 \Rightarrow diam(\bar{G}) \leq 3$.
2. Let T and T_1 be two sub-forests of a graph G with $V(T) = V(T_1) = V(G)$. Suppose that $|E(T_1)| > |E(T)|$ then show that there exists an edge $e \in E(T_1) \setminus E(T)$ such that $T + e$ is also a forest.
3. Let Q_n be the hypercube graph on $\{0, 1\}^n$. What are $\alpha(Q_n), \alpha'(Q_n), \beta(Q_n), \beta'(Q_n)$?
4. Let G be an $X - Y$ bipartite graph such that $|N(S)| > |S|$ for all $\emptyset \subsetneq S \subset X$. Let $e \in E$. Show that there is a matching M containing e .

Some notations :

- G is assumed to be a finite simple graph everywhere.
- If G is a graph, \bar{G} (the complement) is defined as the graph with vertex set $V(G)$ and edge set $\{(u, v) : (u, v) \notin E(G)\}$.
- d_G is defined as the usual graph metric when all edge weights are taken to be 1 and $diam(G) := \max d_G(u, v) : u, v \in V(G)$.
- Q_n is the hypercube graph on $\{0, 1\}^n$ i.e., $V = \{0, 1\}^n$ and $x \sim y$ is x and y differ exactly in one-coordinate i.e., $|\{i : x_i \neq y_i\}| = 1$ where $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_n)$.

- $\alpha'(G)$ - Maximum independent edge set ; $\beta'(G)$ - Minimum edge cover.
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